In order to test the accuracy of the work of computation as well as to test the convergence of the series, it is sometimes advisable to find the value of the definite integral with $r=2a$ after its value has been found with $r=a$. The work involved in this test is usually not great as the work that has been done when $r=a$ is made use of when $r=2a$.

To show the rapid convergence of (1) and (2) the two following simple examples will suffice:

$$\int_0^{\frac{1}{2}\pi} \frac{\cos(3x/2)\sin x}{\sin x}dx = \int_0^{\frac{1}{2}\pi} \cos\frac{3x}{2}dx = \tfrac{2}{3}\sqrt{\tfrac{1}{2}}.$$ Here $m=\frac{3}{2}$, $n=1$. Taking

$r=3$, we have $$\int_0^{\frac{1}{2}\pi} \frac{\cos(3x/2)\sin x}{\sin x}dx = \frac{\pi}{12}(1+\sqrt{\tfrac{1}{2}}) + \frac{3\pi^2\sqrt{\frac{1}{2}}}{6^3.4} + \frac{7\pi^4\sqrt{\frac{1}{2}}}{30.6^4.2.4!} = .47141.$$

Similarly, $$\int_0^{\frac{1}{2}\pi} \frac{\sin(3x/2)\sin x}{\sin x} = \frac{\pi}{12}(2+3\sqrt{\tfrac{1}{2}}) + \frac{3\pi^2}{6^3.4}(1+\sqrt{\tfrac{1}{2}}) + \frac{\pi^4}{30.6^4.2.4!}$$

$\times(\frac{27}{4})(1+\sqrt{\frac{1}{2}})=1.13807$, both results being correct to five decimal places.

Also solved by G. B. M. Zerr.

DIOPHANTINE ANALYSIS.

137. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Prove that all multiply perfect numbers of multiplicity n having only n distinct primes are comprised in $n=2, 3, 4$.

Solution by JACOB WESTLUND, Ph. D., Purdue University, Lafayette, Ind.

If $p_1, p_2, \dots, p_n$ are the distinct prime factors of a number of multiplicity n, we must have $n < \prod_i^{1,n} \frac{p_i}{p_i-1}$, and hence $n < \frac{2}{1}.\frac{3}{2}.\frac{5}{4}.\frac{7}{6}\dots\frac{2n-1}{2n-2}$. But this is impossible when $n>4$, as seen by induction. For we have

$$(n+1)1.2.4.6\dots2n = n.1.2.4.6\dots2n + 1.2.4.6\dots2n.$$

Now if $n.1.2.4.6\dots(2n-2) > 2.3.5.7\dots(2n-1)$, it follows that

$$(n+1)1.2.4.6\dots2n > 2.3.5.7\dots(2n-1)2n + 1.2.4.6\dots2n, \text{ or}$$
$$(n+1)1.2.4.6\dots2n > 2.3.5.7\dots(2n+1) - 2.3.5.7\dots(2n-1) + 1.2.4.6\dots2n.$$

Hence $(n+1)1.2.4.6\dots2n > 2.3.5.7\dots(2n+1)$. For $n=5$ we have

$$5 > \tfrac{2}{1}.\tfrac{3}{2}.\tfrac{5}{4}.\tfrac{7}{6}.\tfrac{9}{8} = \tfrac{315}{64}.$$

Hence for all values of $n>4$ we have $n > \prod_i^{1,n} \frac{p_i}{p_i-1}$, which proves the theorem.